Nassau County Interscholastic Mathematics League

Contest # 3 **2002-2003**

No Calculators

Problems 13-14. Time limit: 10 minutes.

13. The determinant of a 2 by 2 matrix, denoted $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$, is defined by $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.

Find all x for which $\begin{vmatrix} 2x & 1 \\ x & x \end{vmatrix} = 3$

14. In triangle ACD, point B is on $\overline{AC}$ and point E is on $\overline{CD}$ such that $\overline{DB}$ bisects $\angle$ADC and $\overline{EB}$ bisects $\angle$DBC. If AB = 4, BC = 6, DB = 8 and CE = 3, find length AD.

Problems 15-16. Time limit: 10 minutes.

15. One leg of a right triangle is 20 and the other two sides are either consecutive even integers or consecutive odd integers. Find the length of the hypotenuse of the triangle.

16. How many different 5-letter "words" can be made by rearranging the letters of the word LLAMA? [note: "word" does not necessarily mean an English word. It can be any combination of letters in any order]

Problems 17-18. Time limit: 10 minutes.

17. Find the ordered pair of positive integers (a,b) for which $a^2 - b^2 = 37$.

18. Find the smallest positive integer which leaves
- a remainder of 1 when divided by 2,
- a remainder of 3 when divided by 4,
- a remainder of 5 when divided by 6,
- a remainder of 7 when divided by 8,
- and a remainder of 9 when divided by 11 [read carefully].

Answers.

13. $-1, \frac{3}{2}$ 14. $\frac{14}{3}$

15. 101 16. 30

17. $(19,18)$ 18. 119